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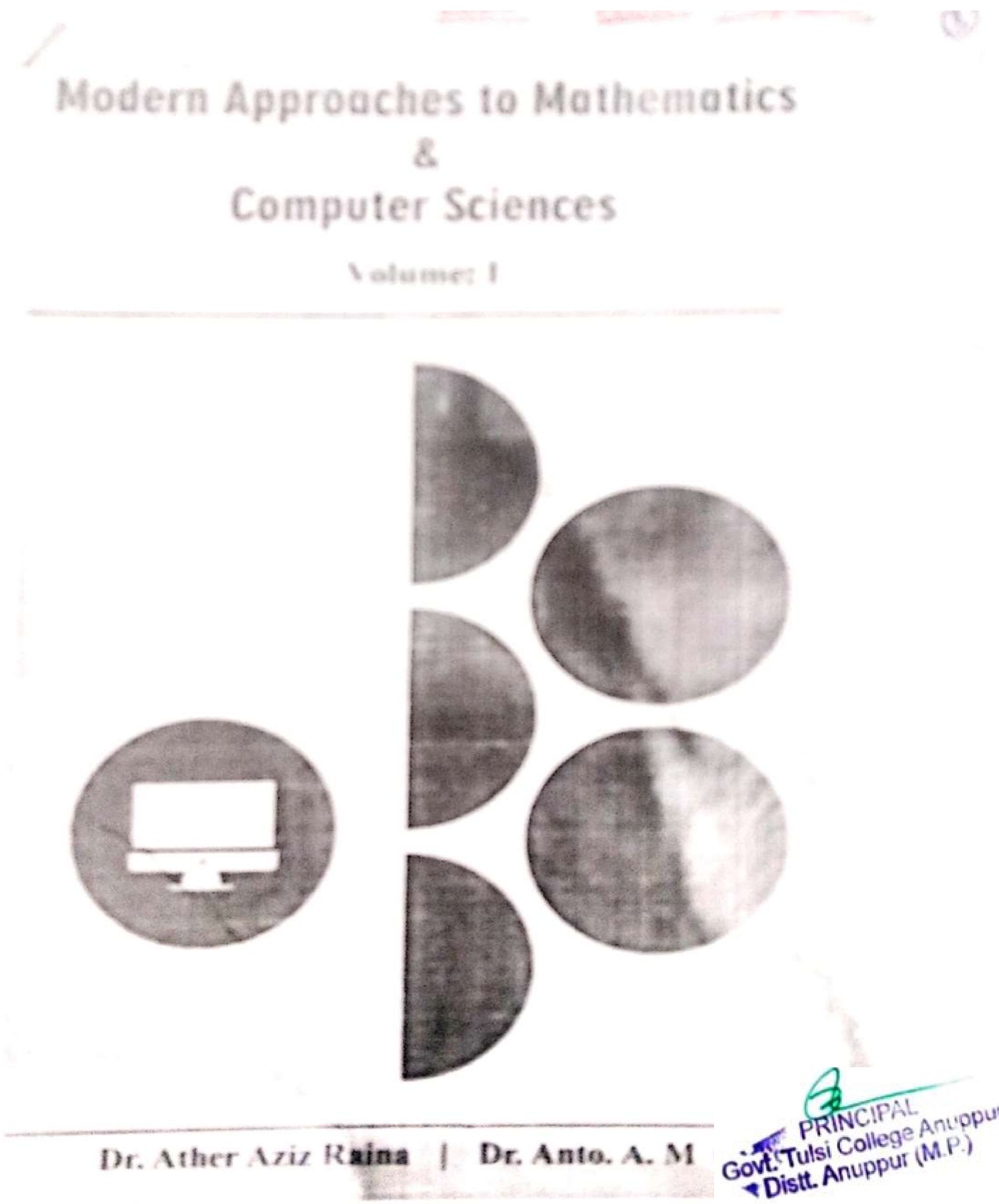
E-mail: hegtdcano@mp.gov.in ☎9893076404

Generalized Pseudo-Protective Curvature Tensor of Quasi Para-Sasakian Manifolds



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...ized Pseudo-Projective Curvature Tensor of Qu... Para-Sasakian Manifolds

Giteshwari Pandey, Dept. of Mathematics, Govt.Tulsi College, Anuppur, M.P

Abstract

The object of the present paper is to generalize pseudo-proje... curvature tensor of quasi-para-Sasakian manifold with the help of a generalized (0,2) symmetric tensor Z introduced by Mantica and Suh... Various geometric properties of generalized pseudo-projective curv... tensor of quasi-para-Sasakian manifold have been studied. It is shown t... generalized pseudo-projectively ϕ-symmetric quasi-para-Sasakian man... is an η-Einstein manifold.

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1. Introduction

I.Sato ([20], [21]) defined the notions of an almost paracon... Riemannian manifold and a paracontact Riemannian manifold and stud... several properties of these manifolds. Adati and Miyazawa [1] investig... some properties of paracontact Riemannian manifolds. They stud... hypersurfaces of paracontact Riemannian manifolds. Paracontact struct... is an analogue of the almost contact structure ([3], [19]) and is closely rel... to almost product structure. An almost contact manifold is always o... dimensional but an almost paracontact manifold could be even dimensi... as well. Later, in 1985, the detailed study of almost paracontact geometry w... carried out by Kaneyuki and Williams [9] and then it was continued by m... other authors. Zamkovoy [26] started the systematic study of alm... paracontact metric manifolds. The notion of quasi Sasakian manifol... introduced by Blair [2], unifies Sasakian and Cosymplectic manifol... A quasi-Sasakian manifold is a normal almost contact metric manifold wh... fundamental 2-form $\Phi(X,Y) = g(X,\phi Y)$ is closed. Quasi-Sasakian manifolds can be viewed as an odd dimensional counter part of Kaehler structures. These manifolds were studied by several authors such as [5], [6], [25] and others. Olszak [17] studied normal almost contact metric manifolds of dimension 3. He gave certain necessary and sufficient conditions for an almost contact metric structure to be normal. Also, various curvature properties of such structures and normal almost contact metric structures on a manifold of constant curvature are studied. De et al. [7] studied 3-dimensional quasi-Sasakian manifolds with semi-symmetric non-metric connection. Quasi-Sasakian manifolds are studied by many others. In the continuation of these studies, Kupeli Erken [11] introduced quasi-para-Sasakian manifold and investigated basic properties and general curvature identities of quasi-para-Sasakian manifolds. He proved that if a quasi-para-Sasakian manifold is of constant curvature K then $K \leq 0$ and (i) if $K = 0$, the manifold is paracosymplectic, (ii) if $K < 0$, the quasi-para-Sasakian structure of the manifold is obtained by a homothetic deformation of para-Sasakian structure. Three dimensional quasi-para-Sasakian manifolds satisfying certain curvature conditions are also studied by Kupeli Erken [10].

On the otherhand, Mantica and Suh [12] introduced a new tensor Z and a new kind of Riemannian manifold that generalize the concept of both pseudo Ricci-symmetric manifold and pseudo projective Ricci-symmetric manifold. They proved that the Z-tensor is the general notion of the Einstein gravitational tensor in general theory of Relativity. Such a new class of manifolds with Z-tensor is named pseudo Z symmetric manifold [12] and denoted by $(PZS)_n$. Also, they studied various properties of $(PZS)_n$, especially focusing the cases with harmonic curvature tensors giving the conditions of closeness of the associated one-form.

The study of pseudo-projective curvature tensor has been a very attractive field for investigations from differential geometric point of view in the past many decades. A tensor field $\overline{P}$ was introduced and studied by Bhagwat Prasad [18] in 2002 on a Riemannian manifold of dimension n, which includes projective curvature tensor P as follows

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$$P(X,Y)U = aR(X,Y)U + b[S(Y,U)X - S(X,U)Y] - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,U)X - g(X,U)Y]$$

This tensor field $\bar{P}$ referred to as pseudo projective curvature ... In 2011, H.G. Nagaraja and G. Somashekhara [16] extended p... projective curvature tensor in Sasakian manifolds. Subsequently, ... researchers performed a study of pseudo projective curvature tensor ... number of directions, such as [8], [13], [14] and others.

Motivated by these studies, we generalize pseudo projective cur... tensor of quasi para Sasakian manifold with the help of a new gener... (0,2) symmetric tensor Z introduced by Mantica and Suh [12]. The pa... organized as follows. After preliminaries in section 3, we introd... generalized pseudo projective curvature tensor and studied some cur... identities of it. In section 4, it is shown that a generalized pseudo projec... semi symmetric quasi para Sasakian manifolds an η Einstein manif... generalized pseudo projectively Ricci semi symmetric quasi para Sa... manifolds an η Einstein manifold have been proved in section 5. In se... 6, it is shown that a generalized pseudo projectively ϕ-symmetric quasi p... Sasakian manifold is an η Einstein manifold.

2. Preliminaries

An $(2n+1)$ dimensional smooth manifold M^{2n+1} has an al... para contact structure (ϕ, ξ, η) if it admits a tensor field ϕ of type (1,1... vector field ξ and a 1-form η satisfying the following conditions ([10], [1...

$$\phi(\xi) = 0,$$
$$\eta(\phi X) = 0,$$
$$\eta(\xi) = 1,$$
$$\phi^2(X) = X - \eta(X)\xi.$$

Distribution $D: p \in M^{2n+1} \to D_p \subseteq T_pM^{2n+1}$: $D_p = \ker\eta = \{...$ $T_pM : \eta(X) = 0\}$ is called paracontact distribution generated by η.

If a manifold M^{2n+1} with (ϕ, ξ, η) structure admits a pseudo Riemannian metric g such that

$$g(\phi X, \phi Y) = -g(X,Y) + \eta(X)\eta(Y), \quad (2.6)$$

then we say that M^{2n+1} has an almost paracontact metric structure and g is called compatible. Any compatible metric g with a given almost paracontact structure (ϕ, ξ, η) is necessarily of signature $(n+1, n)$.

Also, if

$$\eta(X) = g(X, \xi) \quad (2.7)$$

and

$$g(X, \phi Y) = d\eta(X,Y), \quad (2.8)$$

where $d\eta(X,Y) = \frac{1}{2}(X\eta(Y) - Y\eta(X) - \eta[X,Y])$

holds then η is a paracontact form and the almost paracontact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ is said to be a paracontact metric manifold.

A paracontact metric manifold is para Sasakian manifold if and only if

$$(\nabla_X\phi)Y = -g(X,Y)\xi + \eta(Y)X, \quad (2.9)$$

for all vector fields X and Y.

If

$$(\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X, \quad (2.10)$$

then the manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ is said to be a quasi-para-Sasakian manifold ([10], [11]). On quasi para Sasakian manifolds following relations holds good

$$\nabla_X\xi = \phi X, \quad (2.11)$$
$$(\nabla_X\eta)Y = -g(X, \phi Y), \quad (2.12)$$
$$L_\xi g = 0, \quad (2.13)$$
$$L_\xi \phi = 0, \quad (2.14)$$
$$L_\xi \eta = 0, \quad (2.15)$$
$$d\eta(X,Y) = -g(X, \phi Y), \quad (2.16)$$
$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \quad (2.17)$$
$$R(X,\xi)Y = -R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \quad (2.18)$$
$$S(\phi X, \phi Y) = -2n\, g(\phi X, \phi Y), \quad (2.19)$$

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$$S(X,\xi) = -2n\,\eta(X),$$
$$Q\xi = -2n\,\xi,$$
$$r = -2n(2n+1),$$

for any vector fields X, Y, Z, where Q is the Ricci operator $g(QX,Y) = S(X,Y)$, S is the Ricci tensor and r is the scalar curvature tensor.

$$K(X,\xi) = -1,$$

where K is a sectional curvature of a plane section.

3. Generalized Pseudo-Projective Curvature Tensor of Quasi-para-Sasakian Manifolds

In this section, we give a brief account of generalized pseudo-projective curvature tensor of quasi-para-Sasakian manifold and study the geometric properties of it.

The pseudo-projective curvature tensor of quasi-para-Sasakian manifold M^{2n+1} is given by the following relation [18]

$$\overline{P}(X,Y)U = aR(X,Y)U + b[S(Y,U)X - S(X,U)Y] - \frac{r}{(2n+1)}\left(\frac{a}{2n} + b\right)[g(Y,U)X - g(X,U)Y]$$

Also, the type (0,4) tensor field $'\overline{P}$ is given by

$$'\overline{P}(X,Y,U,V) = a'R(X,Y,U,V) + b[S(Y,U)g(X,V) - S(X,U)g(Y,V)] - \frac{r}{(2n+1)}\left(\frac{a}{2n} + b\right)[g(Y,U)g(X,V) - g(X,U)g(Y,V)],$$

where

$$'\overline{P}(X,Y,U,V) = g(\overline{P}(X,Y)U,V)$$

and

$$'R(X,Y,U,V) = g(R(X,Y)U,V)$$

for the arbitrary vector fields X, Y, U, V.

Differentiating equation (3.1) covariantly with respect to W, we get

$$(\nabla_W\overline{P})(X,Y)U = a(\nabla_W R)(X,Y)U + b[(\nabla_W S)(Y,U)X - (\nabla_W S)(X,U)Y] - \frac{dr(W)}{(2n+1)}\left(\frac{a}{2n} + b\right)[g(Y,U)X - g(X,U)Y]. \quad (3.3)$$

Divergence of pseudo-projective curvature tensor is given by

$$(div\overline{P})(X,Y)U = a(divR)(X,Y)U + b[(\nabla_X S)(Y,U) - (\nabla_Y S)(X,U)] - (div\, r)\left[\frac{a+2nb}{2n(2n+1)}\right][g(Y,U)div(X) - g(X,U)div(Y)]. \quad (3.4)$$

But

$$(divR)(X,Y)U = (\nabla_X S)(Y,U) - (\nabla_Y S)(X,U). \quad (3.5)$$

From equations (3.4) and (3.5), we have

$$(div\overline{P})(X,Y)U = (a+b)[(\nabla_X S)(Y,U) - (\nabla_Y S)(X,U)] - (div\, r)\left[\frac{a+2nb}{2n(2n+1)}\right][g(Y,U)div(X) - g(X,U)div(Y)]. \quad (3.6)$$

Definition 3.1 An almost paracontact structure (ϕ,ξ,η,g) is said to be locally pseudo-projectively symmetric if [23]

$$(\nabla_W\overline{P})(X,Y)U = 0, \quad (3.7)$$

for all vector fields $X, Y, U, W \in T_pM^{2n+1}$.

Definition 3.2 An almost paracontact structure (ϕ,ξ,η,g) is said to be locally pseudo-projectively ϕ-symmetric if [24]

$$\phi^2((\nabla_W\overline{P})(X,Y)U) = 0, \quad (3.8)$$

for all vector fields X, Y, U, W orthogonal to ξ.

Definition 3.3 An almost paracontact structure (ϕ,ξ,η,g) is said to be pseudo projectively ϕ-recurrent if [24]

$$\phi^2((\nabla_W\overline{P})(X,Y)U) = A(W)\overline{P}(X,Y)U, \quad (3.9)$$

for arbitrary vector fields X, Y, U, W.

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